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Reteaching

1.3 *The Algebraic Order of Operations*

◆**Skill A** Evaluating numerical expressions by using the rules for the algebraic order of operations

Recall When several operations occur in the same expression,

- perform all multiplications and divisions in order from left to right and then
- perform all additions and subtractions in order from left to right.

◆ **Example 1**

Evaluate the expression $20 + 14 \div 7 \cdot 2$ by using the algebraic order of operations.

◆ **Solution**

$$\begin{aligned} 20 + 14 \div 7 \cdot 2 &= 20 + \mathbf{2} \cdot 2 && \text{Divide 14 by 7.} \\ &= 20 + \mathbf{4} && \text{Multiply 2 by 2.} \\ &= 24 && \text{Add 20 and 4.} \end{aligned}$$

Thus, $20 + 14 \div 7 \cdot 2 = 24$.

◆ **Example 2**

Evaluate the expression $2 \cdot 2 - 2 \div 2$ by using the algebraic order of operations.

◆ **Solution**

$$\begin{aligned} 2 \cdot 2 - 2 \div 2 &= \mathbf{4} - 2 \div 2 && \text{Multiply 2 by 2.} \\ &= 4 - \mathbf{1} && \text{Divide 2 by 2.} \\ &= 3 && \text{Subtract 1 from 4.} \end{aligned}$$

Thus, $2 \cdot 2 - 2 \div 2 = 3$.

Evaluate each expression.

1. $16 - 8 \div 4$ ______________

2. $8 - 20 \div 5$ ______________

3. $6 + 5 \cdot 3$ ______________

4. $7 - 15 \div 5$ ______________

5. $7 + 45 \div 9$ ______________

6. $10 + 14 \div 7$ ______________

7. $5 \cdot 9 - 4$ ______________

8. $6 \cdot 3 + 2$ ______________

9. $3 + 5 \cdot 2$ ______________

10. $5 + 3 \cdot 4$ ______________

11. $3 \cdot 6 + 4 \cdot 2$ ______________

12. $36 \div 6 + 3$ ______________

13. $8 + 12 \div 4 \cdot 6$ ______________

14. $3 - 1 - 8 \div 4$ ______________

15. $4 \cdot 2 + 10 \div 5$ ______________

16. $3 \div 3 + 3 - 3 \div 3$ ______________

17. $8 \cdot 2 + 4 - 1$ ______________

18. $100 - 10 \cdot 5 \cdot 2$ ______________

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◆ **Skill B** Evaluating numerical expressions that contain parentheses

Recall When several operations occur in the same expression,

- do all the evaluations inside any parentheses,
- evaluate each expression having an exponent,
- perform all multiplications and divisions in order from left to right, and then
- perform all additions and subtractions in order from left to right.

The underlined letters in the phrase below can help you remember the algebraic order of operations.

Please **E**xcuse **M**y **D**ear **A**unt **S**ally

Parentheses
Exponents
Multiplications
Divisions
Additions
Subtractions

◆ **Example 1**
Simplify $(7 + 3) \div 2$.

◆ **Solution**

$(7 + 3) \div 2 = \mathbf{10} \div 2$ Add 7 and 3 inside the parentheses, getting 10.
$= 5$ Divide 10 by 2.

Thus, $(7 + 3) \div 2 = 5$.

◆ **Example 2**
Simplify $4^3 + (15 - 7) \cdot 2$.

◆ **Solution**

$4^3 + (15 - 7) \cdot 2 = 64 + (15 - 7) \cdot 2$ Subtract 7 from 15, which gives 8.
$= 64 + 8 \cdot 2$ Multiply 8 by 2, which gives 16.
$= 64 + 16$ Add 64 and 16, which gives 80.
$= 80$

Thus, $4^3 + (15 - 7) \cdot 2 = 80$.

Evaluate each expression.

19. $10 \div (3 + 2)$ ______________

20. $7 + (6 + 4) \cdot 2$ ______________

21. $5 + (6 + 3) \cdot 2$ ______________

22. $5 \cdot (2 + 3) - 7$ ______________

23. $63 \div 9 + (18 - 5)$ ______________

24. $7 \cdot (6 + 5) - 10$ ______________

25. $2^4 \cdot (4 + 2)$ ______________

26. $5^3 + 6$ ______________

27. $4^3 - 30 \div 2$ ______________

28. $14 + (3^3 - 7)$ ______________

29. $4 \cdot 2^2$ ______________

30. $(3^2 + 4) \cdot 3$ ______________

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Reteaching

2.1 The Real Numbers and Absolute Value

◆Skill A Comparing real numbers

Recall The symbols used to compare two real numbers are stated and explained below.

Symbol	Meaning
$<$	is less than
$=$	is equal to
$>$	is greater than

◆ Example

Use $<$, $>$, or $=$ to compare each pair of numbers.

a. -3 and -5 **b.** $-1\frac{3}{4}$ and -1.75 **c.** -10.2 and 8.3

◆ Solution

a. On a number line, -3 is located three units to the left of 0 and -5 is located five units to the left of 0, so -5 is to the left of -3 and -3 is to the right of -5 on the number line. Thus, $-3 > -5$ and $-5 < -3$.

b. Represent both $-1\frac{3}{4}$ and -1.75 as mixed numbers or as decimals. Then compare them.

When $-1\frac{3}{4}$ is represented as a decimal, it is -1.75, so $-1\frac{3}{4} = -1.75$.

c. Negative numbers are always to the left of 0 on the number line, and positive numbers are always to the right of 0 on the number line.

-10.2 is to the left of 0, and 8.3 is to the right of 0, so $-10.2 < 8.3$ and $8.3 > -10.2$.

Use <, >, or = to compare each pair of numbers.

1. 2.45 and -2.45 ____________

2. $\frac{4}{5}$ and $1\frac{4}{5}$ ____________

3. 2.6 and $2\frac{3}{5}$ ____________

4. -2 and $-\frac{4}{2}$ ____________

5. -11 and -12 ____________

6. $2\frac{2}{7}$ and $-\frac{16}{7}$ ____________

7. 0 and 4.5 ____________

8. 0 and -4.5 ____________

◆Skill B Finding opposites and absolute value

Recall You find the opposite of a positive number by changing its sign from + to −.
You find the opposite of a negative number by changing its sign from − to +.

◆ Example

Find the opposite of each number.

a. -10.2 **b.** $5\frac{1}{4}$

◆ Solution

a. The opposite of -10.2 is 10.2. **b.** The opposite of $5\frac{1}{4}$ is $-5\frac{1}{4}$.

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Find the opposite and the absolute value of each number.

9. 2.33 ______ **10.** $\frac{1}{17}$ ______ **11.** $-\frac{9}{2}$ ______ **12.** $-2\frac{6}{13}$ ______

13. 12.56 ______ **14.** -12.56 ______ **15.** 1200 ______ **16.** -0.13 ______

17. -1356 ______ **18.** $3\frac{99}{100}$ ______ **19.** -22.7 ______ **20.** $100\frac{1}{2}$ ______

◆Skill C Simplifying expressions involving opposites and absolute value

Recall When you simplify an expression, you follow the order of operations. You also take the simplification step by step.

◆ **Example 1**
Simplify $-(-12.8)$.

◆ **Solution**
You need to write the opposite of the opposite of 12.8. This is exactly 12.8.

◆ **Example 2**
Simplify $-|24 - 19|$.

◆ **Solution**

$-\|24 - 19\| = -\|5\|$	Perform the subtraction.
$= -5$	Take the absolute value of 5.

Thus, $-|24 - 19| = -5$.

◆ **Example 3**
Simplify $|-2| \cdot |-9|$.

◆ **Solution**

$\|-2\| \cdot \|-9\| = 2 \cdot 9$	Take the absolute value of –2 and of –9.
$= 18$	Perform the multiplication.

Thus, $|-2| \cdot |-9| = 18$.

Simplify each expression.

21. $-|-2.8|$ ______ **22.** $-|2 + 9|$ ______ **23.** $|-5| \cdot |6|$ ______

24. $-(|-2| + |-2|)$ ______ **25.** $|-2| - |2|$ ______ **26.** $|2.5| \cdot |2.5|$ ______

27. $|2 + 8| - |2 + 3|$ ______ **28.** $|2 + 3| \cdot |2 + 3|$ ______ **29.** $(2 + 6) - |6 - 2|$ ______

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Reteaching
2.3 Subtracting Real Numbers

◆Skill A Subtracting real numbers by using a number line

Recall Subtraction is the inverse of addition.

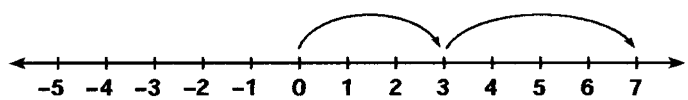
To subtract one real number from another on the number line, start at 0 and move to the position of the first number. From that point, move the number of units indicated by the second number but in the opposite direction of the sign of the number.

◆ **Example**

Subtract: $3 - (-4)$.

◆ **Solution**

First, move 3 units to the right. Then, starting at 3, move 4 units to the right rather than to the left. The ending point is at 7.



Thus, $3 - (-4) = 7$.

Find each difference. Show your work.

1. $4 - (-6)$ __________

2. $-8 - (-2)$ __________

3. $-3 - 7$ __________

4. $0 - (-3)$ __________

5. $-1 - 1$ __________

◆ Skill B Subtracting real numbers and finding distance on a number line

Recall To subtract b from a, add the opposite of b to a.

$$a - b = a + (-b)$$

◆ **Example 1**

Find each difference.

$8 - (-3)$ $\quad -2.4 - 4.6 \quad$ $-\frac{2}{5} - \left(-\frac{1}{5}\right)$

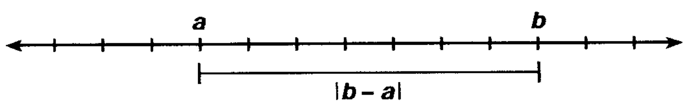
◆ **Solution**

$8 - (-3)$ is the same as $8 + 3$. Thus, $8 - (-3) = 11$.

$-2.4 - 4.6$ is the same as $-2.4 + (-4.6)$. Thus, $-2.4 - 4.6 = -7$.

$-\frac{2}{5} - \left(-\frac{1}{5}\right)$ is the same as $-\frac{2}{5} + \frac{1}{5}$. Thus, $-\frac{2}{5} - \left(-\frac{1}{5}\right) = \frac{3}{5}$.

Recall The distance between a and b on the number line equals $|b - a|$, or $|a - b|$.



◆ **Example 2**

Find the distance between -9 and 7 on the number line.

◆ **Solution**

The distance between -9 and 7 is given by $|-9 - 7|$.

$$|-9 - 7| = |-16|$$
$$= 16$$

Thus, the distance between -9 and 7 is 16 units.

Find each difference.

6. $-38 - (-14)$ ________ **7.** $28 - (-15)$ ________ **8.** $12 - (-28)$ ________

9. $25 - (-23)$ ________ **10.** $-20 - (-18)$ ________ **11.** $-64 - 26$ ________

12. $-1.6 - 4.5$ ________ **13.** $-1.4 - 1.4$ ________ **14.** $-2.8 - (-1.4)$ ________

15. $-3.9 - (-5.6)$ ________ **16.** $7.8 - (-4.7)$ ________ **17.** $-3.6 - 12.2$ ________

Find the distance between each pair of points on the number line.

18. 5 and 15 ________ **19.** -8 and -3 ________ **20.** -4 and 11 ________

21. -18 and -38 ________ **22.** 43 and -12 ________ **23.** 24 and 36 ________

24. -4.3 and -6 ________ **25.** -8.5 and -11.2 ________ **26.** $\frac{1}{2}$ and $-2\frac{1}{4}$ ________

2.6 Adding and Subtracting Expressions

◆ **Skill A** Simplifying expressions with like terms

Recall Like terms can be combined by using the Distributive Property.

◆ **Example 1**
Simplify $(-3x + 2) + (8x - 15)$.

◆ **Solution**
The terms $-3x$ and $8x$ are like terms, and 2 and -15 are like terms.
Rearrange the like terms and then simplify.

$$\begin{aligned}(-3x + 2) + (8x - 15) &= (-3x + 8x) + (2 - 15)\\ &= (-3 + 8)x + (2 - 15)\\ &= 5x + (-13), \text{ or } 5x - 13\end{aligned}$$

Thus, $(-3x + 2) + (8x - 15) = 5x - 13$.

◆ **Example 2**
Simplify $2(-2p + 3q) + (5p + 2q)$.

◆ **Solution**
Apply the Distributive Property, group the like terms, and then simplify.

$$\begin{aligned}2(-2p + 3q) + (5p + 2q) &= 2(-2p) + 2(3q) + (5p + 2q)\\ &= (-4p + 6q) + (5p + 2q) \quad && -4p \text{ and } 5q \text{ are like terms.}\\ & && 6q \text{ and } 2q \text{ are like terms.}\\ &= (-4p + 5p) + (6q + 2q)\\ &= p + 8q\end{aligned}$$

Thus, $2(-2p + 3q) + (5p + 2q) = p + 8q$.

Simplify the following expressions:

1. $(x + 6) + (3x + 3)$ __________

2. $(2m + 4) + (5m - 6)$ __________

3. $(12 - 7y) + (4y + 10)$ __________

4. $(2a + 6b) + (-6a - b)$ __________

5. $(-8x - 13) + (2x - 9)$ __________

6. $(4n + 5) + (3n + 6m)$ __________

7. $(7x - 15) + (23 - 4x)$ __________

8. $(3m + 17) + (-m - 19)$ __________

9. $(4p + 6q - 11r) + (-8p + 6q - 3r)$ __________

10. $(3x - 3y) + (12x - 2y) + (11x - y)$ __________

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◆ Skill B Subtracting expressions

Recall The opposite of $a + b$ is found by using the opposite of a and the opposite of b.

$$-(a + b) = -a + (-b) = -a - b$$

◆ **Example 1**
Simplify the expression $-(3a - 2)$.

◆ **Solution**
Write the opposite of each term.

$$\begin{aligned} -(3a - 2) &= -(3a) - [-(2)] \\ &= -3a - (-2) \\ &= -3a + 2 \end{aligned}$$

Recall To subtract an expression, add its opposite.

◆ **Example 2**
Simplify the expression $(8a - 6b) - (10a + 2b)$.

◆ **Solution**
To subtract $10a + 2b$ from $8a - 6b$, add the opposite of $10a + 2b$.

$$\begin{aligned} (8a - 6b) - (10a + 2b) &= (8a - 6b) + [-(10a + 2b)] \\ &= (8a - 6b) + (-10a - 2b) \\ &= (8a - 10a) + (-6b - 2b) \\ &= -2a + -8b, \text{ or } -2a - 8b \end{aligned}$$

Thus, $(8a - 6b) - (10a + 2b) = -2a - 8b$.

◆ **Example 3**
Simplify the expression $(2 - x) - (-2x)$.

◆ **Solution**

$$\begin{aligned} (2 - x) - (-2x) &= 2 - x + 2x \\ &= 2 + x \end{aligned}$$

Thus, $(2 - x) - (-2x) = 2 + x$.

Simplify the following expressions:

11. $4p - 9p$ __________

12. $(8y - 17) - 9y$ __________

13. $8x - (4 + 11x)$ __________

14. $(m + 3n) - (5m + 7n)$ __________

15. $(13x - 9y) - (4x + 10y)$ __________

16. $(2a + 6b) - (14a - 11b)$ __________

17. $-(3p - 2n - t)$ __________

18. $(12x + 3y) - (6x + 2y - 4)$ __________

19. $(8a - 6b - 3) - (-2a + 4b + 7)$ __________

20. $(8m + 3n - 4) - (3m - 2n) - 8$ __________

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2.7 Multiplying and Dividing Expressions

◆Skill A Multiplying expressions

Recall In the expression x^3, x is called the base, and 3 is called the exponent. The exponent tells how many times the base appears as a factor.

For example, $x^3 = (x)(x)(x)$, $x^1 = x$, and $x^0 = 1$.

◆ Example 1

Simplify the expression $3x \cdot (-5x)$.

◆ Solution

$$\begin{aligned} 3x \cdot (-5x) &= (3x)(-5x) \\ &= (3)(-5)(x)(x) \\ &= -15x^2 \end{aligned}$$

Thus, $3x \cdot (-5x) = -15x^2$.

◆ Example 2

Simplify the expression $4x(2x - 3)$.

◆ Solution

Use the Distributive Property.

$$\begin{aligned} 4x(2x - 3) &= (4x)(2x) - (4x)(3) \\ &= 8x^2 - 12x \end{aligned}$$

Thus, $4x(2x - 3) = 8x^2 - 12x$.

◆ Example 3

Simplify the expression $8x^2 - 3x(x + 1)$.

◆ Solution

Use the definition of subtraction and then use the Distributive Property.

$$\begin{aligned} 8x^2 - 3x(x + 1) &= 8x^2 + (-3x)(x + 1) \\ &= 8x^2 + (-3x)(x) + (-3x)(1) \\ &= 8x^2 + (-3x^2) + (-3x) \\ &= 5x^2 - 3x \end{aligned}$$

Thus, $8x^2 - 3x(x + 1) = 5x^2 - 3x$.

Simplify the following expressions. Use the Distributive Property if needed.

1. $(-2x)(11x)$ ____________ **2.** $5(4x^2) - 2(3x^2)$ ____________

3. $-2(x^2 + x)$ ____________ **4.** $(2x - 3x^2)6$ ____________

5. $x(x + 4)$ ____________ **6.** $6.3x(x - 8)$ ____________

7. $(2x + 10)(5x)$ ____________ **8.** $-7x(5 - x)$ ____________

9. $6x^2 - x(8x + 2)$ ____________ **10.** $-3x^2 - 4x(2 - x)$ ____________

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◆Skill B Dividing expressions

Recall When you divide an expression by a number, you must divide each term in the numerator by that number.

◆ **Example 1**

Simplify the expression $\frac{6a + 24}{6}$.

◆ **Solution**

Use the Distributive Property.

$\frac{6a + 24}{6} = \frac{6a}{6} + \frac{24}{6}$ Write a sum.

$= a + 4$ Divide each numerator by 6.

Thus, $\frac{6a + 24}{6} = a + 4$.

◆ **Example 2**

Simplify the expression $\frac{8(3x - 9)}{6}$.

◆ **Solution**

Use the Distributive Property.

$\frac{8(3x - 9)}{6} = \frac{24x - 72}{6}$

$= \frac{24x}{6} - \frac{72}{6}$ Write a difference.

$= 4x - 12$ Divide each numerator by 6.

Thus, $\frac{8(3x - 9)}{6} = 4x - 12$.

Simplify the following expressions. Use the Distributive Property if needed.

11. $\frac{63a}{-9}$ ________

12. $\frac{-450n}{10}$ ________

13. $\frac{18x + 12}{-6}$ ________

14. $\frac{8k - 12}{-4}$ ________

15. $\frac{70x - 30y}{-5}$ ________

16. $\frac{35x + 210y}{7}$ ________

17. $\frac{3(2x + 4y)}{6}$ ________

18. $\frac{(3x - 8)6}{2}$ ________

19. $\frac{6(5 + 10x)}{10}$ ________

20. $\frac{12(x - 3y)}{4}$ ________

21. $\frac{(5y + 8)3.4}{2}$ ________

22. $\frac{7(-4a - 6)}{1.4}$ ________

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Reteaching

4.1 Using Proportional Reasoning

◆ Skill A Writing ratios

Recall A ratio is a comparison of two quantities by division. Key words such as *to, for, per, out of, in,* and *every* may signal a ratio. You can write a ratio as a fraction.

◆ **Example**
Write a ratio to model the sentence below.
Three out of 10 freshmen take part in athletics.

◆ **Solution**
Write a fraction. Use the part, 3, as the numerator. Use the whole amount, 10, as the denominator. $\frac{3}{10}$

Express each ratio as a fraction in lowest terms.

1. 10 to 25 ______

2. 30 for every 6 ______

3. 18 to 16 ______

4. 6 to 4 ______

5. 6 out of 17 ______

6. 2 to 15 ______

7. 8 in every 10 ______

8. 2 per 5 ______

9. 18 to 1 ______

10. 2000 out of 10,000 people surveyed opposed the bill. ______

11. In our class of 244 students, there were 8 merit scholars. ______

12. The inspector found 3 defective parts in a batch of 500. ______

◆ Skill B Using cross products in proportions

Recall An equation in which two ratios, $\frac{a}{b}$ and $\frac{c}{d}$, are equal is called a proportion.

In the proportion $\frac{a}{b} = \frac{c}{d}$, b and c are the means, and a and d are the extremes.

The cross products of the proportion $\frac{a}{b} = \frac{c}{d}$ are ad and bc.

In any true proportion, cross products are equal. That is, $ad = bc$.

◆ **Example**
Use cross products to find out if the proportion $\frac{3}{5} = \frac{7}{10}$ is true.

◆ **Solution**
The cross products are $3 \cdot 10 = 30$ and $5 \cdot 7 = 35$. Because $30 \neq 35$, the proportion is not true.

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Determine whether each proportion is true. Justify your response by using cross products.

13. $\frac{4}{5} = \frac{7}{8}$ ____________ **14.** $\frac{8}{12} = \frac{6}{9}$ ____________ **15.** $\frac{10}{6} = \frac{20}{12}$ ____________

16. $\frac{20}{100} = \frac{5}{20}$ ____________ **17.** $\frac{14}{16} = \frac{22}{24}$ ____________ **18.** $\frac{15}{18} = \frac{25}{36}$ ____________

19. $\frac{33}{22} = \frac{24}{16}$ ____________ **20.** $\frac{45}{108} = \frac{5}{12}$ ____________ **21.** $\frac{36}{100} = \frac{27}{75}$ ____________

◆Skill B Solving proportions

Recall You can use cross products to write and solve equations.

◆ Example 1

Solve $\frac{10}{12} = \frac{25}{x}$.

◆ Solution

The cross products are $10x$ and $12 \cdot 25$, or 300.

$10x = 300$ Set the cross products equal to one another.

$x = 30$

◆ Example 2

Solve $\frac{x}{1.5} = \frac{3}{2}$.

◆ Solution

The cross products are $2x$ and $3 \cdot 1.5$, or 4.5.

$2x = 4.5$ Set the cross products equal to one another.

$x = 2.25$

Solve each proportion.

22. $\frac{6}{7} = \frac{5.4}{b}$ ____________ **23.** $\frac{a}{5} = \frac{2}{10}$ ____________ **24.** $\frac{18}{p} = \frac{9}{25}$ ____________

25. $\frac{5}{6} = \frac{20}{r}$ ____________ **26.** $\frac{15}{8} = \frac{k}{32}$ ____________ **27.** $\frac{t}{2.4} = \frac{7}{8}$ ____________

28. $\frac{d}{16} = \frac{3}{4}$ ____________ **29.** $\frac{3}{1} = \frac{m}{5}$ ____________ **30.** $\frac{2}{s} = \frac{8}{36}$ ____________

31. $\frac{8}{1.6} = \frac{e}{2}$ ____________ **32.** $\frac{180}{9} = \frac{60}{n}$ ____________ **33.** $\frac{4}{90} = \frac{12}{z}$ ____________

34. A basketball player makes 3 out of 4 free throws. Predict how many successful shots that she will make if she attempts 24 free throws in

one game. ______________________________

Reteaching
8.1 Laws of Exponents: Multiplying Monomials

◆ Skill A Understanding exponents and powers

Recall In the expression 2^4, 2 is called the base, and 4 is called an exponent. The exponent tells how many times to multiply the base by itself. Thus, $2^4 = 2 \cdot 2 \cdot 2 \cdot 2 = 16$. 16 is the fourth power of 2.

◆ Example 1
Find the value of 3^5.

◆ Solution
The base 3 is multiplied by itself 5 times.

$$3^5 = 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 = 243$$

◆ Example 2
Find the value of 5^1.

◆ Solution
Any number raised to the power of 1 is the number itself. Thus, $5^1 = 5$.

Recall When the base is 10, the exponent also tells you how many zeros to place after 1 in the answer.

◆ Example 3
Find the value of 10^3.

◆ Solution
The base 10 is multiplied by itself 3 times. The value of 10^3 is 1 followed by 3 zeros. Thus, $10^3 = 1000$.

Find the value of each expression.

1. 5^4 ________ **2.** 9^2 ________

3. 10^6 ________ **4.** 2^7 ________

5. 8^5 ________ **6.** 3^6 ________

7. 6^1 ________ **8.** 10^4 ________

9. 12^2 ________ **10.** 1^8 ________

◆ Skill B Using the Product-of-Powers Property to simplify expressions

Recall If x is any number and m and n are any positive integers, then: $x^m \cdot x^n = x^{m+n}$.

◆ Example
Simplify.

◆ Solution

$$2^3 \cdot 2^4 = 2^{3+4}$$
$$= 2^7$$

Thus, $2^3 \cdot 2^4 = 2^7$.

Simplify each product. Then find the value of the expression.

11. $3^5 \cdot 3^4$ ______

12. $2^5 \cdot 2^2$ ______

13. $10^5 \cdot 10^3$ ______

14. $5^6 \cdot 5^1$ ______

15. $8^4 \cdot 8^6$ ______

16. $4^3 \cdot 4^4$ ______

◆ Skill C Using the Product-of-Powers Property to multiply monomials

Recall To multiply two monomials, multiply the constants and multiply the variables with the same base.

◆ Example
Simplify $(3m^4n)(-2m^2n)$.

◆ Solution

$$(3m^4n)(-2m^2n) = (3 \cdot -2)(m^4 \cdot m^2)(n \cdot n)$$
$$= -6m^6n^2$$

Simplify each product.

17. $(5a^2)(3a^3)$ ______

18. $(-7cd^2)(3c^2)$ ______

19. $(-s^3t)(-5t^4)$ ______

20. $(6p^5)(4p^2q^3)$ ______

21. $(m^3n^2)(4m^2n^2)$ ______

22. $(a^2b^3)(2b^2c^2)(3a^4)$ ______

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8.2 Laws of Exponents: Powers and Products

◆ Skill A Raising a power to a power

Recall In the expression $(2^3)^2$, 2^3 is raised to the second power. Because 2^3 is used as a factor two times, $(2^3)^2 = 2^3 \cdot 2^3 = 2^6$. When you simplify a power of a power, the exponents are multiplied.

◆ **Example 1**
Find the value of $(3^3)^2$.

◆ **Solution**

$$(3^3)^3 = 3^{3 \cdot 3} = 3^9 = 19{,}683$$

Recall If x is any number and m and n are any positive integers, then $(x^m)^n = x^{mn}$.

◆ **Example 2**
Simplify each expression.
a. $(b^2)^5$ **b.** $(2q^2)^4$

◆ **Solution**

a. $(b^2)^5 = b^{2 \cdot 5} = b^{10}$

b. $(2q^2)^4 = (2^{1 \cdot 4})(q^{2 \cdot 4}) = 2^4q^8 = 16q^8$

Find the value of each numerical expression. Simplify each algebraic expression.

1. $(2^3)^4$ ______ **2.** $(3^2)^3$ ______

3. $(y^4)^3$ ______ **4.** $(m^5)^2$ ______

5. $(2s^3)^2$ ______ **6.** $5(r^5)^2$ ______

7. $(3c^6)^3$ ______ **8.** $(n^3)^d$ ______

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◆ Skill B Raising a monomial to a power

Recall When a monomial is raised to a power, raise each term to that power. $(ab)^2$ means $(ab)(ab)$. Thus, $(ab)^2 = ab \cdot ab = (a \cdot a)(b \cdot b) = a^2b^2$.

If x and y are any numbers and n is a positive integer, then $(xy)^n = x^ny^n$.

◆ Example
Simplify $(3b^2c^3)^2$.

◆ Solution
$(3b^2c^3)^2 = (3^2)(b^2)^2(c^3)^2 = 9b^4c^6$

Simplify each expression.

9. $(10^3)^2$ ________

10. $(2y^3)^5$ ________

11. $(6x^4)^3$ ________

12. $(8q^3)^3$ ________

13. $(c^2d^2)^4$ ________

14. $(9mn^5)^2$ ________

15. $4(e^4f)^3$ ________

16. $(2p^5r^3)^4$ ________

◆ Skill C Finding powers of -1

Recall All even powers of -1 are equal to 1.
All odd powers of -1 are equal to -1.

◆ Example
Simplify each of the following:
a. $(-c)^3$ **b.** $(-2p)^4$

◆ Solution
a. $(-c)^3 = (-1 \cdot c)^3 = (-1)^3(c)^3 = -1(c^3) = -c^3$
b. $(-2p)^4 = (-2)^4(p)^4 = 16p^4$

Simplify each expression.

17. $(-3y^3)^2$ ________

18. $(-gh^4)^5$ ________

19. $(ab^2)^3(-ab^3)$ ________

20. $(-2c^2d^3)^4(-3cd^2)^3$ ________

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Reteaching
8.3 Laws of Exponents: Dividing Monomials

◆ **Skill A** Using the Quotients-of-Powers Property

Recall In a quotient of powers, one power is divided by another. $\frac{2^5}{2^3}$ means $\frac{2 \cdot 2 \cdot 2 \cdot 2 \cdot 2}{2 \cdot 2 \cdot 2}$, so $\frac{2^5}{2^3} = 2 \cdot 2 = 4$.

◆ **Example 1**
Find the value of $\frac{3^7}{3^3}$.

◆ **Solution**
$\frac{3^7}{3^3} = \frac{3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3}{3 \cdot 3 \cdot 3} = 3 \cdot 3 \cdot 3 \cdot 3 = 81$

Recall In the example above, the power of the quotient is the difference between the powers of the numerator and the denominator.

If x is any number except 0 and m and n are any positive integers, where $m > n$, then $\frac{x^m}{x^n} = x^{m-n}$.

◆ **Example 2**
Simplify the expression $\frac{20z^8}{4z^3}$.

◆ **Solution**
$\frac{20z^8}{4z^3} = 5(z^{8-3}) = 5z^5$

Find the value of each numerical expression. Simplify each algebraic expression.

1. $\frac{7^5}{7^2}$ ______________________

2. $\frac{2^9}{2^4}$ ______________________

3. $\frac{8d^8}{2d^2}$ ______________________

4. $\frac{5m^4}{10m}$ ______________________

5. $\frac{10^{15}}{10^6}$ ______________________

6. $\frac{-36r^9}{4r^5}$ ______________________

7. $\frac{-12c^{10}}{-2c^3}$ ______________________

8. $\frac{9g^8}{-g}$ ______________________

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◆ Skill B Finding the quotient of monomials

Recall To divide monomials, divide the constants and the variables with the same base.

◆ **Example**

Simplify $\frac{12a^6b^4}{-3a^4b^3}$.

◆ **Solution**

$$\frac{12a^6b^4}{-3a^4b^3} = \left(\frac{12}{-3}\right)(a^{6-4})(b^{4-3}) = -4a^2b$$

Simplify each expression.

9. $\frac{x^2y^5}{xy^3}$ ______________________

10. $\frac{p^7q^5r^2}{p^3q^4}$ ______________________

11. $\frac{-30g^9h^8}{5g^3h^6}$ ______________________

12. $\frac{24y^8z^5}{-32y^6z}$ ______________________

13. $\frac{-9s^{12}t^9}{-3s^8t^6}$ ______________________

14. $\frac{8.4(b^2c^3)^3}{2b^3c^4}$ ______________________

◆ Skill C Finding the power of a fraction

Recall If n is a positive number and a and b are numbers, where $b \neq 0$, then $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$.

◆ **Example**

Simplify $\left(\frac{25d^4e^6}{5d^2f}\right)^3$.

◆ **Solution**

$$\left(\frac{25d^4e^6}{5d^2f}\right)^3 = \left(\frac{5d^2e^6}{f}\right) = \frac{125d^6e^{18}}{f^3}$$

Simplify each expression.

15. $\left(\frac{2r^3}{n}\right)$ ______________________

16. $\left(\frac{-35c^2m}{5}\right)^3$ ______________________

17. $\left(\frac{-20c^3}{(-2c)^2}\right)$ ______________________

18. $\left(\frac{d^9e^{12}}{(d^2e^3)^2}\right)^2$ ______________________

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Reteaching

8.4 Negative and Zero Exponents

◆ Skill A Understanding negative and zero exponents

Recall There is a pattern between exponents and powers of the same base.

As the exponents decrease by 1, the value of the power decreases by a factor of $\frac{1}{2}$.

2^4	2^3	2^2	2^1
16	8	4	2

◆ Example 1

What is the value of 2^0?

◆ Solution

Add 2^0 to the table above and complete the pattern.

2^4	2^3	2^2	2^1	2^0
16	8	4	2	?

The value of the power decreases by a factor of $\frac{1}{2}$, so $2^0 = 1$.

This pattern will hold true for any base that is a nonzero number, x, or $x^0 = 1$.

◆ Example 2

What is the value of 2^{-1}?

◆ Solution

Add 2^{-1} to the table above and complete the pattern.

2^4	2^3	2^2	2^1	2^0	2^{-1}
16	8	4	2	1	?

The value of the power decreases by a factor of $\frac{1}{2}$, so $2^{-1} = \frac{1}{2}$.

If x is any number except zero and n is any integer, then $x^{-n} = \frac{1}{x^n}$.

Evaluate each expression.

1. 4^0 ______________ **2.** 5^{-2} ______________

3. 8^0 ______________ **4.** 4^{-1} ______________

5. 3^{-3} ______________ **6.** 1^{-2} ______________

7. 5^{-3} ______________ **8.** 4^{-3} ______________

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◆ **Skill B** Simplifying expressions containing negative and zero exponents

Recall To add two integers with the same sign, add their absolute values and keep the common sign. To add two integers with different signs, subtract their absolute values and use the sign of the number with the greater absolute value. Subtraction is the same as adding the opposite.

◆ **Example 1**
Simplify the expression $y^{-5} \cdot y^3$.

◆ **Solution**
To multiply powers of the same base, add the exponents.
$y^{-5} \cdot y^3 = y^{-5+3} = y^{-2}$

◆ **Example 2**
Simplify the expression $\frac{m^4}{m^7}$.

◆ **Solution**
To divide powers of the same base, subtract the exponents.
$\frac{m^4}{m^7} = m^{4-7} = m^{-3}$

◆ **Example 3**
Simplify the expression $c^{-3} \cdot c^0$.

◆ **Solution**
This expression represents a product of powers of the same base, so the product is found by adding the exponents. Thus, $c^{-3} \cdot c^0 = c^{-3+0} = c^{-3}$. Alternatively, $c^0 = 1$ because any base to the zero power equals 1. A factor multiplied by 1 is itself. Thus, $c^{-3} \cdot c^0 = c^{-3} \cdot 1 = c^{-3}$.

Simplify each expression.

9. $a^3 \cdot a^{-5}$ ____________

10. $c^2 \cdot c^{-7}$ ____________

11. $\frac{y^3}{y^6}$ ____________

12. $\frac{m^{-3}}{m^6}$ ____________

13. $p^8 \cdot p^0$ ____________

14. $q^0 \cdot q^{-5}$ ____________

15. $x^{-8} \cdot x^{-3}$ ____________

16. $z^{-5} \cdot z^8$ ____________

17. $\frac{t^{-5}}{t^{-10}}$ ____________

18. $5^{-3} \cdot 5^8$ ____________

19. $x^5 \cdot x^{-3} \cdot x^{-7}$ ____________

20. $3^3 \cdot 3^{-10} \cdot 3^6$ ____________

21. $\frac{t^{-5} \cdot t^5}{t^3}$ ____________

22. $\frac{4^7}{4^{-3}}$ ____________

23. $5^3 \cdot 5^0 \cdot 5^{-1}$ ____________

24. $a^2 \cdot a^{-5}$ ____________

25. $\frac{r^{10} \cdot r^{-2}}{r^5}$ ____________

26. $\frac{2^{10} \cdot 2^{-10}}{2^{10}}$ ____________